

Developmental Mathematics Instructional Support Manual

**Prepared in collaboration with Front Range Community College and Landmark
College, through a Lumina grant over the 2005-2007 academic years.**

Identified Goal

The goal of this handbook is to train and support faculty and build disciplinary communication and consistency by defining a philosophy for teaching developmental-level students at Front Range Community College and outlining practical methods based on that philosophy.

Using this Manual

It is the opinion of the authors that an honest review of this material will most likely occur if workshops are planned to discuss specific parts of the manual. For example, a workshop could center around the Philosophical Principles explained in this manual. To focus the discussion, all participants might be asked to review the Principles as well as a lesson plan from, say, the MAT 030 course specific section. Possible discussion questions could include:

1. How does this lesson plan address the Philosophical Principles?
2. What is the lesson plan missing?
3. How are instructors addressing the Principles in their own lesson plans?

Workshops can be quite informal and still maintain their effectiveness on those in attendance. A math department may decide to include a small discussion during just the last 20 minutes of an already-planned meeting in order to spark conversation. That said, formally arranged workshops have a better chance of including more adjunct instructors. Therefore, math departments are encouraged to hold both formal workshops as well as the impromptu roundtable discussions.

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I. Guiding Philosophy and Principles

Philosophy

Front Range Community College’s Developmental Education Department encompasses English, Math, Reading, and Study Skills content areas, and offers courses that meet students where they are while preparing them for the rigor of college transfer-level academic curricula.

FRCC Developmental Education program is based on a “strengths” model, focusing and building on the strengths of the students and the knowledge they bring into the classroom, rather than working from a “deficit” model, which would start from a perspective that the students are “deficient” and need remediation to “fix” them.

The developmental-level class is a complex and diverse group; therefore, in order to better define our philosophy, we have first identified the students who comprise a typical developmental classroom. While the following is not meant to be an exhaustive list, these students include:

- Students who have scored low on the placement test
 - In other words, this group includes students who may have adequate content skills but lack adequate test taking skills. Standardized tests may not reflect their ability in the content area but rather their ability to successfully navigate standardized tests.
 - At FRCC, Students take the Accuplacer test when they enroll. The following are a breakdown of placements based on those scores.
 - Arithmetic scores between:
 - 24 and 56 place into MAT 030.
 - 57 and 120, or Elementary Algebra scores below 45 (if the test prompts them to take the Elementary Algebra test) place into EMAT 060.
 - Elementary Algebra scores between:
 - 45 and 60 place into MAT 090.
 - 61 and 84 place into MAT 106.
- “Non-traditional” students
 - This group includes students who have been out of school for a while—sometimes as long as 20 years or more.
- English language learners
 - This group includes students who are coming out of secondary and post-secondary ESL programs.
 - This group includes students who communicate well but don’t necessarily have strong literacy in their primary language.
 - This group includes students who speak English but who grew up in homes where English was not the language spoken.
- Students who are successful in other areas of their lives

- We believe it is necessary to stress that we may be seeing students at their least comfortable, and they may have real success in other areas outside of academics. This group includes successful firefighters, construction workers, bakers, accountants, business owners, etc.
- Students with learning disabilities
 - This group includes students with diagnosed *and* undiagnosed LD- which might look like: the tough guy student, the rowdy student, the rebellious student, the most articulate talker in the room, the extremely shy student, the really bright student whose produced work does not reflect her demonstrated or undemonstrated intellect, etc.
- Disenfranchised students
 - This group includes students with years of negative educational experiences; who do not feel comfortable at school; who don't care too much about school; who sign up for class and don't come often; who come to school for class, leave when it's over, and do not get to know others or feel connected to the experience; who do not put high value on grades or achievement; who feel like victims of their grades and teachers; who do not feel in control of their learning.
- First Generation College Students
 - This group includes students who may not be supported in their attempts to get an education. Perhaps they risk alienation from their home community by going to college. They may not be familiar with academic discourse.

With these kinds of students in mind, we believe that our philosophy needs to be based on helping these students not only increase their skills, but also increase their sense of being a part of mainstream college life.

To accomplish this, we believe that students in developmental classes should be given *authentic college learning experiences* that prepare them to be successful in future college courses, granting them access to a higher education, and providing them with life enhancing opportunities.

II. Theoretical Framework

III. Overarching Principles and Methods

a. Overview

b. Philosophical Principles

- The majority of the students in developmental classes are college -able, and come to developmental courses with profound and useful funds of knowledge. It is the developmental educator's obligation to validate and utilize these funds of knowledge as a vital part of our course content.

- Developmental students are capable of engaging in college level curriculum that includes explicit instructions in skills and strategies.
- Collaboration and active involvement creates more opportunity for learning. The content, learning resources, assessment procedures, and grading reflect these beliefs by incorporating a variety of activities in addition to traditional lecture methods.
- Using a variety of learning styles in developmental pedagogy creates more opportunity for learning. The content, learning resources, assessment procedures, and grading reflect these beliefs by incorporating a variety of activities in addition to traditional lecture methods.
- Developmental courses are most effective when they are related to the life experiences of the students.
- Developmental students are capable of becoming active members of the college community.
- Developmental students learn most when they reflect and focus on understanding themselves as learners, developing learning strategies that work for them, and understanding how they can use college resources to meet their goals.
- Developmental students learn most when they are explicitly taught to access processes, tools, and other resources that foster their growth as strong, independent learners.

c. General Principles of Instruction

- i. Identify the relevant skills and knowledge students exhibit, and account informally for how these factors may enable or impede their learning.
 - Begin instruction at the point of break down
 - Assess progress frequently and modify instruction in response
 - Focus on underlying factors rather than surface errors
 - Identify academic strengths and areas of weakness
 - Provide frequent, objective feedback on progress
- ii. Provide explicit instruction in relevant skills and strategies
 - Teach and practice fundamental skills until they become automatic
 - Teach and model skills and procedures explicitly
 - Break tasks and skills into sub-skills, staged procedures and sequential processes

- Provide regular reinforcement and review
 - Disclose and teach students about the hidden curriculum
- iii. Create a student-centered classroom
- Incorporate a variety of language domains and processing modes, such as expressive language and elaborative rehearsal, structured and independent practice, and experiential and hands-on learning, into instruction and review
 - Set clear expectation for participation, and provide support for students in meeting them
 - Draw on the relevant knowledge students possess when introducing and developing new skills, ideas and themes
 - Create conditions that help students to experience success through their action.
- iv. Design instruction with care
- Craft and customize assignment directions to reduce ambiguity
 - Focus on organizational elements and time planning, and identify after-hours resources available to students for cases of unanticipated difficulties
 - Design and use standardized forms to streamline feedback, and to address course objectives consistently
 - Limit the focus of directive feedback with students, as well as the length and scope of their individual conferences
- v. Address the diversity of learning styles
- Create a strength-based instructional context by helping students to reflect on what they are doing that is supporting their learning
 - Incorporate visual-spatial, kinesthetic and tactile modalities whenever these modalities enhance learning
 - Regularly vary the means by which students demonstrate mastery of the material
 - Provide alternative procedures and strategies for performing tasks
 - Value and incorporate alternative modes of perceiving and thinking when it is clear that these modes will support the achievement of academic goals and objectives.
- vi. Base instruction and assessment of on clearly stated objectives
- Explicitly identify class agendas, learning goals, means and standards for assessment
 - Link agendas and goals to specific learning objectives
 - Link assessment to specific learning goals
 - Hold students accountable for understanding goals and objectives

- vii. Foster self-understanding and self-efficacy
 - Build reflection into the learning process
 - Be explicit about how learning takes place and breaks down
 - Incorporate process reflection into tasks and assignments
 - Link self-understanding with strategy use
 - Support internalization of the student's locus of control

- viii. Work with a developmental mindset
 - Be direct, concise, clear and consistent in communicating with students
 - Encourage students to set relevant personal goals, and assist them in monitoring their progress
 - Integrate themes and ideas of emerging relevance into the curriculum
 - Avoid doing anything for students that they can do for themselves
 - Guide students to reflect on and draw from their own available resources and problem-solving abilities
 - Assume that college students are adults, and provide assistance as needed in helping students to understand and meet the expectations of a post-secondary learning context
 - Give students failing grades when failing grades are warranted, guide them in reflecting on what they might do differently and assist them in leaving previous failures behind

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d. Mathematics Instruction for All Learners

i. What we can do?

In general, as instructors our influence on learners is limited to about 50% of those characteristics that directly affect the ability of students to succeed. Those areas of influence can be categorized as:

Learning Accommodation – what we can do to account for different learning styles and needs. For example, students present themselves in the classroom as visual learners, auditory learners, or kinesthetic learners. Or they may have difficulty with fluid reasoning, or short-term memory.

Testing Provisions – what we can do to better assess students' achievements. Tests aren't always a good measure of what students understand. Instead, other methods of assessment might better gauge a student's understanding such as allowing: collaborative activities, adjusting the timing of assessment activities, or changing the length of assessment activities.

Math Study Skills – what we can teach to students about learning how to learn. Studies show that students benefit from direct instruction in:
time management, note-taking skills, active learning, homework skills, reading math texts, error analysis, test taking skills, controlling math anxiety.

ii. **What to do:**

(some general learning differences or problems students might have and what to do)

1. Processing speed difficulties look like:

Student has difficulty with speed and accuracy of copying, identifying symbols, taking tests, and reading comprehension.

This is what you can do:

- Allow time for copying for the board. Stop writing. Tell the class when they do or do not need to copy.
- Teach “chunking” and “tracking” to facilitate copying for the board and textbook.
- Provide handouts: procedure charts, partial notes, homework sheets with space allocated for solutions.
- Teach meeting time goals by having occasional smaller quizzes, maybe just 3 questions, with a shorter time. So instead of 12 questions in 30 minutes, have 2 questions in 5 minutes. Teach students to “go on” in a test.

2. Visual-spatial processing difficulties can look like:

Reversing and transposing numbers, difficulty discriminating between factors, numbers, variables, keeping numbers in columns, keeping handwriting aligned on paper, understanding and discriminating geometric forms and number arrays.

This is what you can do:

- Use larger paper or turn paper on its side. Use graph paper. Provide direct instruction in paper organization for visual clarity. use white space on handouts to create clarity. Utilize color to help discriminate between easily confused symbols. Highlight operation signs.

3. Language Processing and Memory difficulties can look like:

Difficulty reading math text that requires rapid switches from word symbols to math symbols, difficulty remembering steps, difficulty with application problems, may have difficulty receiving lecture material, hard time learning of math facts.

This is what you can do:

- Provide opportunities for reverbaling. Provide visual support of oral instruction and directions. Have students teach the concept. Use study partners. Micro-unit instruction. Provide lots of

practice and review. Connect the last lesson with today's work. Have students re-cap the main points of lesson. Have students use flashcards and videos. Plan for review days and review main ideas and concepts.

4. Long-term retrieval difficulties look like:

Difficulty remembering mathematics facts, difficulty remembering facts that need to be applied to new learning. May perform well on quizzes but fails tests.

This is what you can do:

- Micro unit instruction. Break processes down into smaller manageable and memorable pieces.
- Teach “overlearning.” What students might think is remembering might just be recognition.
- Have students use flashcards and videos.
- Give frequent reviews always tying in the most recent with the previous material.
- Use multi-sensory instruction. Some students might remember a diagram longer than prose.
- Provide lots of varied practice.
- Utilize concept note cards. Just the process of creating the cards requires students to prioritize and closely examine the material even if the cards are not used by submitted on the test day.
- Plan for review days and review main ideas and concepts.

5. Fluid Reasoning difficulties can look like:

Difficulty understanding and applying a formula to a new problem, difficulty relating concepts with symbolic language, poor abstract reasoning and difficulty generalizing from one experience to a new situation, not being able to apply a concept.

This is what you can do:

- Utilize the lesson plan components: 1. review; 2. introduce new material; 3. guided practice with feedback; 4. independent practice.
- Have shorter sessions, in other words, teach smaller chunks of information and how they relate to each other.
- Introduce one concept at a time to avoid confusion.
- Have students create 5x8 concept cards.
- Provide additional support outside of class.
- Encourage verbalization of understanding.
- Explicitly connect problems to examples.
- Group students to make it easier to get around to assist them.
- Make sure you assign a variety of problems to do in class that reflect all the problems you demonstrated.

iii. **Classroom management for Attentional Issues**

There are always moments when attention waivers, but we can have some control.

- Develop routines and consistent procedures: provide clear agenda and objectives for each class; preview class activities and goals; make transitions and connections explicitly; review and summarize frequently; establish regular patterns for tests and due dates; vary within the routines periodically for novelty and stimulus.
- Provide diverse modalities for learning: present information visually as well as auditorily; provide opportunities for hands-on learning; incorporate expressive as well as receptive activities; allow movement and physical activity. Some techniques:
 - Visual: agendas, graphics, templates, film, videos, photos, artifacts
 - Hands on learning: role play, field work, use of manipulatives and models
 - Expressive: talk, work in pairs or small groups so everyone has a chance to talk; write –whenever student glaze over, give them some problems to do.
- Hold student accountable for deadlines: provide clear, step-by-step directions; clearly define all expectations; help students break tasks into manageable steps; assess interim assignments
- Help students maintain focus: vary activities within each class; cue students to verbalize what they are doing and why; curtail and redirect tangents and interruptions; develop monitoring and cueing strategies for individual students.
- Plan ahead! If you don't have a plan, students will create their own agenda!

iv. **Structure of a lesson**

Studies show that how we structure our lesson affects learning outcomes.

- Put the plan on the board.
- Review yesterday's concepts/skills.
- Connect yesterday to today.
- Teach the new concept and related vocabulary.
- Give students guided practice.
- Give students time for independent practice.
- Evaluate the learning accomplished.
- Other suggestions on how to structure lessons are included in course-specific guidelines.

v. **Best Practices**

For ALL learners

(from “Mathematics Interventions for Adolescents with Learning Disabilities.” Paula Maccini and Charles Hughes. Learning Disabilities Research and Practice 12(3)

1. Design lesson content to include the following: discriminate among similar problem-types; use a wide range of examples; separate confusing elements; have students meet critical performance before advancing to the next stage.
2. Provide immediate feedback.
3. Micro-unit your presentation of materials so student can master one small piece of information at a time.
4. Establish an independent baseline of knowledge – do not assume student can remember previously learned material.
5. Provide structured practice during class time.
6. Provide detailed models for student to use when doing independent work.
7. Have structured worksheets available for student.
8. Include a combination of modalities in your presentation.
9. Integrate periodic “booster sessions” or “spiral days” to review what was learned in the lessons of the last few days.

vi. **The Art of Questioning**

How we ask questions can influence the type of thinking involved in responding. Examples:

Process – What method did you use on that problem?

Review – What are some of the things you’ve learned so far in trigonometry?

Motivate – What are some four-sided geometric figures you can name?

Diagnose – How did you get that answer?

Challenge – What evidence do you have for thinking that?

Question – What questions does this information leave unanswered?

Transfer – How could you use that result in this new situation?

Guide – Do you remember a problem similar to this one?

Evaluate – How do you think your method would work on this next problem?

Interact – Judy, do you agree with what Brian said?

Explore – Can you find a pattern in those numbers?

IV. Curriculum

a. Overview

Our curriculum aims to explicitly teach and empower students to call into practice specific strategies for completing college level work.

For instance, when students are confronted with a problem, either in a math classroom or in a real life context, student will be able to examine it and say, “This is what I need to do to solve this problem.” In other words they will have enough practice in strategy development that they will be able to automatically call up an appropriate strategy to solve whatever problem they encounter.

Overall Learning Outcomes:

After completion of the developmental course sequence, students will:

- Have acquired the basic language and learning skills required as a foundation for further academic work and the workplace.
 - Have developed strategies, processes, and competencies that allow them to read, write, problem-solve, study, communicate orally, and learn effectively at a college level.
 - Have developed self-understanding and self-management strategies for learning.
- (adapted from Landmark College)

b. Course Specific Learning Objectives

i. MAT 030 Fundamentals of Mathematics

State Curriculum: Includes basic operations with whole numbers and decimals with an introduction to fractions.

1. Learning Outcomes

Arithmetic Skills

- Demonstrate a basic understanding of the structure of our number system
- Demonstrate competency in the usage of whole numbers, decimals and commonly used fractions (without calculators)
- Demonstrate the ability to communicate using the vocabulary of arithmetic

Number Sense

- Demonstrate the ability to estimate a reasonable solution without performing the actual calculations

Study Skills

- Demonstrate an understanding of good study techniques for college students
- Identify differences between studying mathematics and other disciplines

Math in Life

- Demonstrate an ability to apply these math tools to everyday life
- Demonstrate an ability to convey an understanding of these math tools

Positive Math Attitude

- Identify past experiences which may have led to a negative attitude toward mathematics

- Move toward an internal locus of control concerning future mathematics education

2. Topics Covered

- a. Place value
- b. Addition, subtraction, multiplication, and division of whole numbers, decimals, and commonly used fractions
- c. Rounding and estimation
- d. Exponential notation
- e. Order of operations
- f. Applications with whole numbers, decimals and fractions
- g. Prime factorization
- h. Math study skills

3. Embedded Teaching Strategies

- a. Extensive use of manipulatives
- b. Writing to examine personal math attitudes
- c. Writing to demonstrate conceptual understanding
- d. Extensive use of group work and collaborative learning
- e. Students can create their own set of flash cards, giving them an additional modality for learning these facts
- f. Assigned projects to encourage the application of these tools to everyday life

4. Sample Lesson Plan

Course: Math 030Lesson: Course Introduction, Sections 1.1-1.2Date: 03/07/06Lesson Outcome: Understanding of Whole Numbers, Addition of Whole NumbersSpecial Equipment/Materials Required:

I. Introduction

Names, hobbies, work, intended job, dream job.

A. Attention Getter

- How much money is typically spent on a presidential election campaign?

B. Performance Outcomes and Purpose

- Place Value
- Writing Numbers in Words
- Using the Number Line
- The Addition Algorithm
- Applications with Addition

C. Relate to Prior Knowledge

- Ask the students to talk about where they see whole numbers.
- Explain that they are constantly doing math.

II. Development

A. Key Points

- Understand the number in front of you.

B. Mode(s) of Instruction

- Short lecture
- Solving problems as a class/groups
- Next Item
- **RECAP**

C. Activities

- Explain the naming convention for the place values.
- Explain that there [a 7 in the hundreds place] means that this means [7 hundred] plus the other numbers.
 - Guided Practice
- Write a number in standard notation and then that number in words.
- Write a number in words and then that number in standard notation.
 - Guided Practice
- Explain the greater than and less than symbols
 - Guided Practice
- Show how to add using the number line.
- Show the addition algorithm and emphasize carrying the one.
 - Guided Practice
- Discuss various applications from the book.

- D. Guided Practice
 - Students identify the digit in the: tens place, hundreds...
 - Students identify the place value of a digit.
 - In groups: Students convert numbers between standard notation and words.
 - Students place $<$ or $>$ between two numbers.
 - Students work many examples of adding numbers.
 - Groups: Applications

 - E. Independent Practice – “Show your work. Then check your answers in the back of the book.”
 - Assign many odd-numbered exercises so students can check their work.
 - Even if it counts for very few points, collect this homework so students will actually do it!
- III. Consolidation
- A. Assessment
 - Question/answer period

 - B. Review
 - **Review key points**

 - C. Transfer
 - “Bring to the next class: An experience at work or another class where you had to use any of the topics discussed today.”

 - D. Evaluation
 - Students write what they thought was the most important thing learned; what was the hardest?

5. Subtract:

$$\begin{array}{r} 82,545 \\ - 58,482 \\ \hline \end{array}$$

6. Multiply:

$$\begin{array}{r} 4,568 \\ \times 700 \\ \hline \end{array}$$

7. (4 points) Estimate the product by rounding each factor to the nearest hundred: (Show your work.)

$$\begin{array}{r} 438 \\ \times 284 \\ \hline \end{array}$$

8. Multiply:

$$\begin{array}{r} 438 \\ \times 284 \\ \hline \end{array}$$

For Problems 9, 11, and 12 please write your answers in remainder form where applicable.

9. Divide: $94 \div 16 =$

10. (2 points) Check your answer for problem number 9. Show your work.

11. Divide: $17 \overline{)5219}$

12. Divide: $35 \overline{)2431}$

13. Solve: $25 + x = 84$ $x =$

14. Evaluate: 4^3

15. Simplify: $2^2 + 32 \div 4$

16. Simplify: $(22 + 34) \div 4 + 3$

17. Find the average of 56, 67, 57, and 68.
18. A bag of apples weighs 12 lb. A bag of oranges weighs 9 lb. Find the total weight of 13 bags of apples and 12 bags of oranges combined.
19. A lab technician pours 413 mL of an acid solution into 7 beakers, pouring the same amount in each. How much of the solution is in each beaker?
20. (4 points) Which of the following are true? (Select all that apply.)
- a. $12 < 17$
 - b. $a - b = b - a$ for any numbers a and b
 - c. $a + b = b + a$ for any numbers a and b
 - d. $2^3 > 3^2$
21. (4 points) What do we mean when we say math follows a sequential learning pattern?
22. (4 points) Define each of the following:

a. Difference –

b. Quotient –

c. Divisor –

d. Product -

7. Find the reciprocal of

a) $\frac{4}{7}$

b) 5

8. Reduce $\frac{54}{90}$ to lowest terms.

9. Simplify

a) $\frac{0}{3} =$

b) $\frac{3}{0} =$

Multiply

10. $\frac{5}{9} \times \frac{3}{25}$

11. $42 \times \frac{5}{6}$

12. $\frac{6}{7} \times 4 \times \frac{14}{27} \times \frac{3}{8}$

Divide

13. $\frac{3}{8} \div \frac{5}{12}$

14. $\frac{1}{8} \cdot \frac{12}{21} \div \frac{6}{14}$

15. Solve: $\frac{4}{5} \cdot t = \frac{3}{10}$

16. Write 360 as a product of prime numbers.

17. Solve: $\frac{5}{14} = \frac{?}{56}$

18. $\frac{3}{4} \div \frac{1}{2} \times \frac{2}{3}$

19. A recipe for piecrust calls for $\frac{3}{4}$ cup of flour. A chef is making $\frac{1}{2}$ of the recipe. How much flour should the chef use?

20. I want to parcel out 120 pounds of dry dog food into $\frac{3}{4}$ -pound packages. How many packages can I make?

MAT 030 Fundamentals of Mathematics

Name _____

Test 3a
Fall 2005

-
- Notes and references are not allowed on this exam. A calculator is **not** allowed on this exam.
 - Each problem is worth 5 points.
 - The time limit for the exam is 50 minutes.
 - When you are finished, turn in the exam and all scratch paper.
 - Please show your work and circle your answer.
-

1. Find the LCM of 12, 18, and 27.

For questions 2 – 6 perform the indicated operations. Be sure to simplify all fractions.

2. $\frac{1}{6} + \frac{3}{8}$

3. $\frac{3}{8} + \frac{5}{12} + \frac{2}{15}$

4. $\frac{7}{12} - \frac{3}{8}$

5. $\frac{5}{9} + \frac{7}{24} - \frac{5}{18}$

6. $\frac{1}{3} + \frac{3}{4} \cdot \frac{5}{6}$

7. A pizza was divided between three people. One received $\frac{5}{12}$ of the pizza and the second received $\frac{1}{4}$ of the pizza. What part of the pizza was left for the third person?

8. Which of these fractions is greater? $\frac{4}{5}$ or $\frac{7}{9}$
Show work to support your answer.

9. Convert $4\frac{3}{7}$ to fraction notation. (an “improper” fraction)

10. Convert $\frac{29}{6}$ to a mixed numeral.

11. Divide. Write a mixed numeral for the answer. $57 \overline{)8368}$

12. Describe in your own words how to rewrite a mixed numeral in fraction notation (as an “improper fraction”). Please use complete sentences.

For problems 13 – 18 write your answer as a mixed numeral.

13. $23\frac{5}{8} + 17\frac{5}{6}$

14. $37\frac{1}{6} - 19\frac{4}{9}$

15. $2\frac{2}{5} \cdot 3\frac{3}{4}$

16. $3\frac{1}{3} \div 2\frac{1}{12}$

17. $4\frac{1}{2} + 3\frac{3}{4} - 1\frac{2}{3}$

18. $\left(\frac{2}{3}\right)^2 + 2\frac{2}{9} \cdot \frac{3}{5}$

19. I want to parcel out 60 pounds of dog food into $1\frac{2}{3}$ pound packages. How many packages can I make?

20. Greta had 5 gallons of paint. She used $2\frac{2}{3}$ gallons to paint her family room and $1\frac{1}{4}$ gallons to paint her kitchen. How much paint did Greta have left?

6. Which of the following are true? (Select all that apply.)
- a) You must line up the decimal point when adding decimal numbers.
 - b) You must line up the decimal point when multiplying decimal numbers.
 - c) You must line up the decimal point when subtracting decimal numbers.
 - d) You must move the decimal to the end of the divisor when dividing decimal numbers.
7. Find the value of x : $0.5 \cdot x = 1.75$
8. $6.3 + 9 + 1.78$
9. $6.007 - 0.04$
10. $8 - 0.53 + 0.6$
11. $4.58 \cdot 0.015$
12. a) $7.023 \div 0 =$
b) $0 \div 7.023 =$
13. Divide and round your answer to the nearest hundredth if necessary: $17.23 \div 0.12$

14. Perform the operations in the proper order to evaluate: $(0.3)^2 \div 3 + 0.2 \cdot 0.04$
15. A beef roast weighing 2.3 pounds costs \$3.59 per pound. What is the total cost of the roast to the nearest cent?
16. Helga traveled 85.44 miles on 3.2 gallons of gasoline. How many miles per gallon did her car achieve?
17. On a given day, a charity received donations of \$195.00 and \$35.50. They also paid utility bills of \$67.65 and \$54.92. After paying the utility bills, how much money was left from the day's donations?
18. A group of college students rented 3 condominiums at a ski resort for a weekend. Each condo's rent is \$310 for the weekend. The group totals 12 students. If the cost is split evenly, how much will each one pay?

19. Helga is buying a new car. If she pays cash, the price is \$22,300. However, she can finance it over 48 months, paying \$518.50 per month. If she pays by the installment plan, how much **more** does she pay for the car?

20. Explain how to quickly (in your head) multiply by a power of ten.

ii. MAT 060 Pre-Algebra**State Curriculum:**

1. Learning Outcomes

Arithmetic/Pre-Algebra Skills

- Demonstrate competency in the usage of real numbers
- Demonstrate ability with fraction operations (Including simple fractions involving variables)
- Demonstrate the ability to evaluate expressions
- Demonstrate the ability to simplify basic algebraic expressions using the commutative, associative, and distributive properties
- Demonstrate the ability to solve basic equations

Mathematics for Career and Life skills

- Demonstrate an ability to work with percents and proportions
- Demonstrate an understanding of simple interest applications

Math in Life

- Demonstrate an ability to apply these math tools to everyday life
- Demonstrate an ability to convey an understanding of these math tools

2. Topics Covered

3. Embedded Teaching Strategies

- a. Extensive use of manipulatives
- b. Writing to demonstrate conceptual understanding
- c. Extensive use of group work and collaborative learning
- d. Assigned projects to encourage the application of these tools to everyday life

4. Sample Lesson Plans

Course: Math 060Lesson: Section 7.1Date: 8/28/02Lesson Outcome: Understand How Fractions, Decimals, and Percents Are EquivalentSpecial Equipment/Materials Required: Small m&ms packages, SmartBoard

The following is a handout for each student. The instructor will guide the students along with the table displayed on the SmartBoard. Values in the table will be blank for the students' copies. They will fill in the table. The instructor can highlight the numbers and make the text white. Then, during class, the instructor can change the text color to reveal the filled-in table.

In this project, you will be building on your knowledge of fractions and decimals to find percentages in your bag of m&ms. The process is as follows:

1. Open your bag, and count how many of each color you have. Record each number in the "Number" column. Add these numbers and record the total sum.
2. Write the fraction of the total that each color represents. For example, if you have 3 blues and a total of 25 m&ms, blue represent $3/25$ of the total.
3. Convert each fraction to a decimal and round to the nearest *thousandth*.
4. Convert each decimal to a percent by moving the decimal place two digits to the right and attaching a % sign.

	Number	Fraction	Decimal	Percent
Blue	7	$7/23$.304	30.4%
Red	3	$3/23$.130	13.0%
Orange	5	$5/23$.217	21.7%
Yellow	3	$3/23$.130	13.0%
Green	5	$5/23$.217	21.7%
Brown	0	$0/23 = 0$.000	0.0%
TOTAL	23	$23/23 = 1$	1.000 ?	100%

DISCUSSION

1. Add all the fractions. What should your total be?

2. Add all the decimals. What do you **expect** you total to be? How might rounding affect the outcome?
3. Add all the percents. What should your total percent be?
4. How did your results compare with other people in the class?
5. Why aren't all bags of m&ms exactly the same?
6. How do you think they choose how many of each color to include?
7. How do you think a new color affects the ratio and percent of the other colors?

*Think About: How could you use the m&ms to make a circle graph?

5. Sample Projects

Project 1**DUE: Thursday, October 26, 2006****15 Points**

This assignment is designed to give you practice solving some real-life applications using the tools we discuss in class. This project is worth 15 points. You can get help from the instructor on any aspect of the project. You may discuss the project with others, but do not use the same numbers as anyone else (except for Part I).

The report should be 1 – 2 pages in length. **The report must be typed on a computer.** Grading will primarily be on mathematical content, but you are also responsible for making the report correct and easily read. **Spelling and grammar do count!** A good suggestion is to write the report so that you would understand it five years from now. **The format should be that of a report with complete sentences, paragraphs and section headings.** It is particularly helpful if your section headings match those below. You do not need to show your work. Rather, you are required to **explain** what you are doing.

PART I: Order of Operations

Simplify:

$$\frac{\{8 - 3(-2)^2 \div (2 - 8) - 2[3(-7 + 9)]\} + \frac{4}{3}}{-\frac{4}{6} \left[3 - 2\left(\frac{1}{2}\right) - 12\left(\frac{1}{2}\right)^2 \right]}$$

It is a good idea to work this by hand first. Then, instead of showing your work on the report, **describe in words** each step that you took.

PART II: House Dimensions

1. Measure the dimensions of rooms on one floor of your house/apartment and find the [approximate] square footage of area of that floor. (This can be just a rough measurement between the walls.)
2. Write an expression that gives the volume of that floor of your house, where you know the area of the floor, and the height of the ceiling is your variable. (Start with the formula: Volume = Floor area*height.) Then plug in the actual height of your ceiling to find the volume of that level of your house/apartment.

PART III: Travel

1. Go to the website <http://www.xe.com/ucc/> and find out how much \$1 U.S. would equal in the foreign currency of a country you would like to visit.
2. Now, without using the website, use that exchange rate to calculate how much foreign currency you would have if you exchanged \$275 U.S. Remember to explain how you made the conversion.

Project 2

DUE: Tuesday, December 5, 2006

15 Points

This assignment is designed to give you practice solving some real-life applications using the tools we discuss in class. This project is worth 15 points. You can get help from the instructor on any aspect of the project. You may discuss the project with others, but do not use the same numbers as anyone else.

The report should be 1 – 2 pages in length. **The report must be typed on a computer.** Grading will primarily be on mathematical content, but you are also responsible for making the report correct and easily read. **Spelling and grammar do count!** A good suggestion is to write the report so that you would understand it five years from now. **The format should be that of a report with complete sentences, paragraphs and section headings.** It is particularly helpful if your section headings match those below. You do not need to show your work. Rather, you are required to **explain** what you are doing.

PART I: Recipe Proportion

1. Find a recipe (for food or any other mixture of ingredients) that has at least five different ingredients. Give the original recipe.
2. Imagine that you don't have enough of one ingredient (i.e. You have only 3 cups of flour when the recipe calls for 4 cups.) Adjust the rest of the ingredients so that they are proportional to the limiting ingredient.

PART II: Investments

1. Keep track of the amount of money that you or your household spends during one week.
2. Find the percent of your weekly spending that was spent on things you want (as opposed to things you need).
3. If you decrease your spending on “wants” by 25%, how much will you have left to spend during the week?
4. Take 25% of the dollar amount that you spent on things that you want, and multiply it by 52 (weeks in a year). Imagine that you invested this amount in a mutual fund. Do a search on mutual funds and find the average annual growth rate of any particular fund. (Try Vanguard or Fidelity, as a start.) If you invested your full year's savings from above into your chosen mutual fund, how much money would you have in that account after 20 years? Assume the same annual growth rate for each year. Use the formula: $A = P(1 + i)^t$ NOTE THE EXPONENT!!! YOU WILL NEED A SCIENTIFIC CALCULATOR FOR THIS!!!

Where P = the initial amount that is invested

i = the annual growth rate (as a decimal)

t = number of years the money is invested

A = the final amount after the money has been in the account for t years

PART III: Recipe in Metric

Use either the recipe from Part I or find a new recipe. Convert all of the ingredients into metric measurements. For example, if the recipe calls for 1 cup of water, convert that to the appropriate volume in milliliters.

6. Sample Assessments

This is a two-part test. Part I does not allow calculators. Part II does allow calculators. After students complete Part I, they get Part II.

Pre-Algebra: Math 060

Test #2: Chapters 3 - 5

PART I: NO CALCULATORS

Name: _____ Date: _____

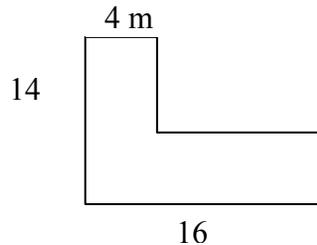
Show your work where applicable. Check your answers. Please simplify all fraction answers. Unless otherwise specified, round decimal answers to the nearest hundredth. Label your answers. CIRCLE OR BOX all answers. Good luck!

Part I: Fifteen no partial credit and five possible partial credit problems.

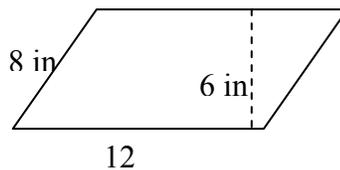
Part II: Five no partial credit problems and three possible partial credit problems.

Problems 1 - 15: 3 Points each; No partial credit

1. Find the perimeter of the figure:



2. Find the area of the shape below:



3. Write an algebraic expression (not and equation), using x as a variable, to represent:

Five times a number plus 5

4. Write the fraction in lowest terms: $\frac{-9}{27}$

5. Multiply and simplify: $\left(10\frac{3}{4}\right)\left(1\frac{1}{2}\right)$

6. Divide and simplify: $\frac{3}{x} \div \frac{3x}{7}$

7. Multiply and simplify: $\left(\frac{3x}{4}\right)\left(\frac{5}{xy}\right)$

8. Add: $\frac{-7}{8} + 3$

9. Subtract: $\frac{x}{2} - \frac{3x}{4}$

10. Subtract: $10\frac{3}{4} - 3\frac{3}{5}$

11. Simplify: $\frac{2}{9} - 4\left(\frac{5}{6}\right)$

12. Simplify: $\frac{1}{3}\left(\frac{3}{5}\right)^2$

13. Solve: $\frac{x}{6} = \frac{3}{8}$

14. Solve: $\frac{2}{3}x - 10 = -5$

15. Solve: $0.8x = 56$

Problems 16 – 22: 5 Points Each

16. A rectangle measures 3.4 cm by 0.5 cm. Find the area of the rectangle.

17. Mr. Chee had 11 packages of lightbulbs. After he used 36 of the lightbulbs, he had 8 left. How many lightbulbs were in each package?

18. A rectangular garden has a length of $10\frac{3}{4}$ ft and a width of $6\frac{1}{2}$ ft. How many feet of edging do you need to buy if you want to put edging around the perimeter of the garden?

19. Add: $\frac{x}{4} + \frac{2}{3}$

20. Which is the larger circle: a circle with an area of 25π square inches, or a circle with a circumference of 18π inches? WHY? Explain your reasoning.

PART II: CALCULATORS ARE ALLOWED

Name: _____ Date: _____

Show your work where applicable. Check your answers. Please simplify all fraction answers. Unless otherwise specified, round decimal answers to the nearest hundredth. Label your answers. CIRCLE OR BOX all answers. Good luck!

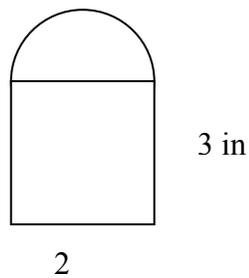
Cylinder: Surface Area = $2\pi \cdot r^2 + 2\pi \cdot rh$
 Volume = $\pi \cdot r^2 h$

Problems 21 – 25: 3 Points Each, No Partial Credit

21. Solve: $2.3x - 4 = -18.35$

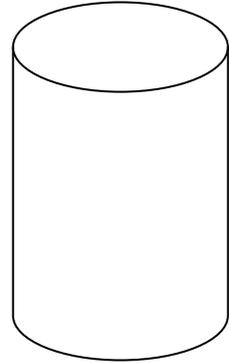
22. Solve: $3.2x + 2.55x = 120 - x$

23. Find the area of the following figure:



24. Simplify: $2.3 - 4.5(3.2 + 1.2)$

25. The following figure has a radius of 12 inches and a height of 20 inches. Find its volume.



Problems 26 - 28: 5 Points Each

26. A UFO left another circular crop circle! If the space ship had a radius of 15 meters, what area of crops were smashed? If there are 30 ears of corn in every square meter of land, how many ears of corn were smashed?
27. If one small can of paint covers 75 square feet, and your outside wall measures 22 feet wide by 16 feet tall, how many cans of paint do you need to buy?
28. You notice that one die from your pair of dice in Monopoly is a cube. If the die is 1.3 centimeters tall, what is the surface area of the die?

iii. MAT 090 Introductory Algebra

State Curriculum: Includes first-degree equations, inequalities, formulas, polynomials, algebraic fractions, factoring polynomials, solving quadratic equations by factoring, and applications. Coordinate geometry, graphing linear equations and inequalities, and systems of linear equations may be included.

1. Learning Outcomes

Principles of Algebra

- Demonstrate an understanding of the difference between evaluating expressions and solving equations.
- Demonstrate the ability to factor polynomials
 - i. Recognize a greatest common factor
 - ii. Recognize that a binomial (or larger polynomial) can be a greatest common factor
 - iii. Factor by grouping

Graphing

- Demonstrate an understanding that a slope is a rate of change
- Graph a line on the coordinate plane
- Find the slope of a line that goes through two particular points

Algebraic Manipulations

- Demonstrate ability to simplify complex fractions
- Find the values of the variable that make a [rational] expression undefined
- Demonstrate an ability to simplify rational expressions

Math in Life

- Demonstrate an ability to apply these math tools to everyday life
- Demonstrate an ability to convey an understanding of these math tools

2. Topics Covered

- First-degree equations and inequalities
- Operations on polynomials
- Factoring of polynomials
- Algebraic fractions (rational expressions)
- Coordinate geometry (plotting points and graphing lines)
- Linear systems of equations

3. Embedded Teaching Strategies

- Build upon previous knowledge: “Operations on rational functions follow the same principles that you already know from fractions.”
- Theme for Solving Equations: “Isolate the variable!”
- Treat terms of a polynomial as entities; add x-terms the way you add apples
- Plotting points can be demonstrated with desks aligned for the x- and y-axes. Students can physically stand in the position of their assigned coordinates.
- Find the slope of a road, stairs, hill, etc., then draw this to emphasize rate of change (for every 6 inches up, you move 8 inches forward).
- Assigned projects to encourage the application of these tools to everyday life

4. Sample Lesson Plan

Course: Math 090Lesson: 1.1-1.3Date: 8/28/02Lesson Outcome: Algebraic expressions; Commutative, associative, distributive; Fractions and factoringSpecial Equipment/Materials Required:

I. Introduction

- A. Attention Getter
- Labor Day BBQ – $\frac{1}{4}$ hamburger for n guests
- B. Performance Outcomes and Purpose
- Algebraic Expressions and equations
 - Commutative Law
 - Associative Law
 - Distributive Law [and Factoring]
 - Prime Factorization
 - Fractions and operations
 - Reducing (simplifying)
- C. Relate to Prior Knowledge
- Who recognizes these terms? THEN WRITE AN EXAMPLE ON THE BOARD

II. Development

- A. Key Points
- FOR TODAY
- B. Mode(s) of Instruction
- Short lecture
 - Solving problems as a class/groups
 - Next Item
 - **RECAP**
- C. Activities
- Parts of an expression: variable, constant (Michelle said coefficient was a constant – she's right), coefficient, terms
 - Translate sentences into expressions and equations – 3-4 examples and then GROUPS
 - Define EQUIVALENT expressions
 - Commutative and Associative for addition and multiplication. NOT subtraction or division
 - Distributive for $a(b+c)$; backwards is factoring; GROUPS
 - Prime number definition and prime factorization
 - $A/a = 1$; $a(1) = a$
 - Fraction multiplication, division, adding, subtracting – COMMON DENOMINATORS; Groups
- D. Guided Practice
- Groups of 4: Translate sentences and expressions back and forth
 - Distribute and factor expressions

- Add, subtract, multiply, and divide fractions

E. Independent Practice – “Look at answers first if it helps you.”

- HW: pg. 7-8: 1-9 odds, 15
 - Pg. 16-17: 33-43, 53, 57-63 odds
 - pg. 24-25: 1-5, 41, 51, 55, 59, 61, 63, 79
 - 27 exercises total.

III. Consolidation

A. Assessment

- Question/answer period

B. Review

- **Review key points**

C. Transfer

- Bring to the next class: An experience at work or another class where you had to use any of the topics discussed today.

D. Evaluation

- Students write what they thought was the most important thing learned; what was the hardest?

5. Sample Projects

Project 1**DUE: Monday, October 15, 2006*****15 Points***

This assignment is designed to give you practice solving some real-life applications using the tools we discuss in class. This project is worth 15 points. You can get help from the instructor on any aspect of the project. You are also encouraged to discuss your project with other students, but do not use the same numbers as anyone else.

The report should be 1 – 2 pages in length. **The report must be typed on a computer.** Grading will primarily be on mathematical content, but you are also responsible for making the report correct and easily read. **Spelling and grammar do count!** A good suggestion is to write the report so that you would understand it five years from now. **The format should be that of a report with complete sentences, paragraphs and section headings.** It is particularly helpful if your section headings match those below. You do not need to show your work.. Rather, you are required to **explain** what you are doing.

Part 1: Discovering Conversions

1. Measure the heights of 3 or 4 friends in **both** inches and centimeters, and fill out a table like the one shown (with more rows, of course):

Name	Height in Inches	Height in Centimeters

2. The pairs of inches and centimeters will give you ordered pairs to plot on the coordinate plane. Use inches as your x-coordinates and centimeters as your y-coordinates. You may either draw this plot or use Excel to plot the points.
3. Use any two of your points to find the slope of a line that goes through the points. In this case, you should write your slope as a decimal.
4. Determine where the line would cross the y-axis.
5. Look up the conversion factor for converting from inches to centimeters, and compare this with your slope. It should be close. Describe what the slope of your line represents.

Part 2: Small Business

3. Imagine that you are selling a product or service (dog houses, oil changes, tax preparation, necklaces, etc.). Choose your product or service, and determine the capital (overhead) cost. For example, you may need tools and instructional books to make dog houses.
4. Now determine (or guess) how much it will cost to make each item or service that you want to sell. For example, in addition to the capital cost for the dog houses, you must use a certain amount of wood and other materials for each dog house.
5. Write a Cost function that includes your capital cost **and** your *variable* cost. (The variable cost will depend on how many goods or services you sell.)

6. Write a Revenue function that represents the amount of money that you receive from buyers. Again, this will *vary* depending on the number of goods or services you sell.
7. Find the minimum number of goods or services that you must sell to Break Even.

Part 3: Dimensions

3. Imagine that you are building a house with a large, open basement. If the length is x feet, the width is $(x-4)$ feet, and the height is $(x-12)$ feet, multiply the factors to make a 3rd degree polynomial function $V(x)$ that represents the volume of the room.
4. Find the volume if $x = 20$ feet.

Project 2

DUE: Wednesday, May 3, 2006

15 Points

This assignment is designed to give you practice solving some real-life applications using the tools we discuss in class. This project is worth 15 points. You can get help from the instructor on any aspect of the project. You may receive help from others on the how-to's for the computing aspects of the project, but do not use the same numbers as anyone else.

The report should be 1 – 2 pages in length. **The report must be typed on a computer.** Grading will primarily be on mathematical content, but you are also responsible for making the report correct and easily read. **Spelling and grammar do count!** A good suggestion is to write the report so that you would understand it five years from now. **The format should be that of a report with complete sentences, paragraphs and section headings.** It is particularly helpful if your section headings match those below. You do not need to show your work. Rather, you are required to **explain** what you are doing.

STEP 1: Hobbies or Home Improvement

Describe an activity or home project that you would like to complete (build a fence, knit a sweater, paint a house, etc.)
Estimate how long it will take you to complete the activity.

STEP 2: Get By with a Little Help from Your Friends

Think of a friend or family member, and estimate how long it would take that person to complete the task. Now calculate how long it would take for both of you to complete the task if you were to work together.

STEP 3: Start a Business

Think of one more person and estimate the time it would take for that person to complete the activity. Now calculate how long it would take for all three of you to complete the task.

Imagine a fourth person and estimate how long it would take that person to complete the activity. Now calculate how long it would take for all four of you to complete the task if you were to work together. Now calculate how long it would take for all four of you to complete ten of these tasks.

***For an A grade, make sure each of the four people works at a different pace. For example, each of the workers Ben, Tom, Adam, and Sam may take 12 hours, 13 hours, 10 hours, and 6 hours, respectively, to finish the task on their own.**

6. Sample Assessment
This is a two-part test.

Introductory Algebra: Math 090

TEST 2
Chapters 2 – 3

Part I: No Calculator

Name: _____ Date: _____

Show your work where applicable. Check your answers. Please simplify all fraction answers. Unless otherwise specified, round decimal answers to the nearest hundredth (two decimal places). CIRCLE OR BOX all answers. Good luck!

Part I: Five no partial credit and 10 possible partial credit problems.

Part II: Five no partial credit and 10 possible partial credit problems.

Problems 1 - 5: 2 points each; no partial credit

1. Determine whether 5 is a solution to $32-x^2=10$

2. Solve: $5x-8=38$

3. Solve: $-(2/3)x=(3/4)$

4. Solve for x: $2x-b=5x$

5. Solve the inequality: $2y-1<5$

12. The sum of three consecutive odd integers is 213. Find the integers.

13. Imagine that you have no more than 72 inches of edging to make an artistic rectangular frame. If you want the length to be twice as long as the width, what range of values can you make the width? Set this problem up as an inequality and solve to get credit.

14. What is the slope of the line between the points (0,4) and (-2,-1)?

15. Sketch and label a rectangular coordinate system and graph the equation $x+2y=6$. You may either plot points or use the concept of the slope-intercept equation.

Part II: Calculators are allowed.

Name: _____ Date: _____

Show your work where applicable. Check your answers. Please simplify all fraction answers. Unless otherwise specified, round decimal answers to the nearest hundredth (two decimal places). CIRCLE OR BOX all answers. Good luck!

Problems 16 - 20: 2 points each; no partial credit.

16. Solve for I: $P=(I/(rt))$

17. Solve: $12+3.5x=16$

18. What is 15% of 80?

19. True or false: When you see a negative number somewhere in an inequality, you should automatically change the direction of the inequality symbol.

21. In which quadrant is the point (2,-3)?

Problems 21 - 30: 4 points each.

21. Solve: $2.4p+7.1=3.3p-10.4+1.6p$

22. Solve and give your answer in interval notation: $9x+3\geq-24$

23. Solve and give your answer in interval notation: $-6y\geq-72+2y$

24. Suppose that your salary went from \$9.25/hour to \$10.25/hour. What is the percent increase?
25. Five-fourths of a number is equal to that number plus 25. What is the number?
26. Solve, graph the solution, and write the solution in interval notation:
 $-1 \leq 4x - 5 \leq 7$
27. Suppose that you want to have at least \$3000 per month take home pay, after the government takes 24% of your salary. At least how much should your monthly salary be so that you can take home at least \$3000 per month after taxes?
28. The heating bill for a house averages \$800 per winter, while the bill for a well-insulated house averages \$575 per winter. What is the percent savings in heating bills as a result of good insulation?
29. Imagine that you are traveling at 15,840 inches per minute! How fast is this in miles per hour?

30. The cost, C , in dollars of a rental truck can be expressed with the equation: $C=0.6x+20$, where x is the number of miles the truck is driven.

a) Find the C -intercept and interpret what this number means.

b) How much does each additional mile cost the renter?

iv. MAT 106 Survey of Algebra

State Curriculum: Emphasizes problem solving with further study of equations, slope, inequalities, systems of equations, polynomials, quadratic equations, rational expressions, rational exponents, radical expressions, graphing and applications.

1. Learning Outcomes

Principles of Algebra

- Demonstrate knowledge and understanding of functions
- Evaluate a function at a given value of x
- Evaluate a function with an arbitrary expression
- Understand that the function takes an input and gives an output
- Demonstrate an understanding of the domain and range

Graphing

- Understand that a zero of a function is the x -intercept
- Graph a function without a calculator
- Graph a system of inequalities without a calculator
- Understand the absolute value function as a piecewise function

Algebraic Manipulations

- Demonstrate the ability to manipulate radicals and rational exponents
- Demonstrate the ability to simplify complex fractions
- Demonstrate the ability to simplify rational expressions

Math in Life

- Demonstrate an ability to apply these math tools to applications
- Demonstrate an ability to convey an understanding of these math tools

2. Topics Covered

- Factoring, rational expressions, and coordinate geometry (Review as needed.)
- Introductory Algebra and Problem Solving
- Functions and Graphing Linear Functions
- Systems of Equations in Two and Three Variables
- Inequalities and Absolute Value
- Polynomial Functions
- Rational Expressions and Equations
- Rational Exponents and Radicals
- Quadratic Equations and Functions

3. Embedded Teaching Strategies

- Solving problems: Encourage students to feel confident in their abilities. Rather than feeling overwhelmed by a large problem, they can break the problem into smaller parts, all of which they can do.
- Build upon previous knowledge: “Graphing linear functions is just like graphing equations in the slope-intercept form.”
- Always check your answer for how reasonable it is.
- Assigned projects to encourage the application of these tools.

4. Sample Lesson Plan

Course: Math 106Lesson: Sections 6.4, 6.5Date: 10/16/06Lesson Outcome: Complex Fractions, Division of PolynomialsSpecial Equipment/Materials Required:

I. Introduction

A. Attention Getter

- What anxieties arise when you hear the word “complex”?
- Explain that Complex Fractions are not really complex in terms of difficulty!

B. Performance Outcomes and Purpose

- Understand that a complex fraction is one where the numerator and denominator are made up of fractions. It’s just like having part (a) and part (b) in an exercise. Tedious, yes. Difficult, no!
- Understand how division of polynomials follows a very similar algorithm to long division of numbers.

C. Relate to Prior Knowledge

- Briefly review adding, subtracting, multiplying and dividing rational expressions (just like fractions).

II. Development

A. Key Points

- Fractions
- Division

B. Mode(s) of Instruction

- Short lecture
- Solving problems as a class/groups
- Next Item
- **RECAP**

C. Activities

- Begin with $\frac{\frac{1}{2} - \frac{1}{4}}{\frac{1}{4} - \frac{1}{8}}$ and explain how you can simplify the numerator and denominator independently, and then just divide.

- Divide: $\frac{\frac{x-y}{x^2}}{\frac{x-3}{xy^3}}$

- Show how you can multiply each term by the LCD for $\frac{x - \frac{2}{y}}{\frac{1}{x} - \frac{3}{y}}$

- Let students simplify this either by simplifying N and D and then dividing, or

$$\frac{x-1}{x+2} \cdot \frac{x-2}{x+3}$$

by multiplying each term by the LCD: $\frac{x-3}{x^2+5x+6}$

- Example: $\frac{x^{-3} + y^{-3}}{xy^{-2}}$
- Division: first show a monomial divided by a monomial: $-30x^3 \div (-5x)$
Show that this can be written as a fraction.
- $(6x^3 - 5x^2) \div (4x^2)$
- Reminder about the long division algorithm: $15 \overline{)225}$
- $(-10x^4 - 5x^3 - 6x - 7) \div (-5x^2)$ Set this up as long division and explain that you need to write BOTH the divisor and the dividend in complete descending order. Explain that the division is finished when the remainder has a degree less than that of the divisor.
- $\frac{x}{x-1}$ Explain that this is NOT $\frac{x}{x} - \frac{x}{1}$!!!
- Check: Dividend = Divisor*Quotient + Remainder
- $\frac{x^3-1}{x-2}$ Write answer as $Quotient + \frac{Remainder}{Divisor}$
- $\frac{5x^2-13x+13}{5x-13}$
- $\frac{x^2-6x+8}{x+4}$ Explain that if there is no remainder it means that $(x+4)$ is a *factor* of the dividend.
- More as time allows.
- Applications as time allows.

D. Guided Practice

- On most examples in class, start working the exercise, and then let students guide you to the correct answer, and/or let the students work in groups to finish the exercise.

E. Independent Practice – “Look at answers first if it helps you.”

- About 40 – 50 odd-numbered exercises.

III. Consolidation

A. Assessment

- Question/answer period

B. Review

- Review key points**

C. Transfer

- Bring to the next class: An experience at work or another class where you had to use any of the topics discussed today.

D. Evaluation

- Students write what they thought was the most important thing learned; what was the hardest?

5. Sample Project

Project 1**DUE: Monday, October 30, 2006*****15 Points***

This assignment is designed to have you explore some of the direct and indirect advantages of having a strong math comprehension. This project is worth 15 points. You can get help from the instructor on any aspect of the project. You are also encouraged to discuss your project with other students, but do not use the same numbers or topics as anyone else (except on Part I).

The report should be 1 – 2 pages in length. **The report must be typed on a computer.** Grading will primarily be on content, but you are also responsible for making the report correct and easily read. **Spelling and grammar do count!** A good suggestion is to write the report so that you would understand it five years from now. **The format should be that of a report with complete sentences, paragraphs and section headings.** It is particularly helpful if your section headings match those below. You do not need to show your work like you do on a test. Rather, you are required to **explain in words** what you are doing.

PART I: Linear Programming for a Small Business

6. Imagine that you and a friend are making rocking chairs and porch swings. Each rocker requires 3 hours of work from you and 2 hours of work from your friend. Each swing requires 2 hours of work from you and 2 hours of work from your friend. You cannot work more than 48 hours per week, and your friend cannot work more than 40 hours per week.
7. Declare your variables and set up the system of inequalities that models the number of chairs and swings that can be made in a week.
8. Graph the system of inequalities, and note any intersection points that lie on the feasible region (the shaded area). The intersection points are called corner points. Don't forget the intersection points on the x- and y-axes. Show your graph in your report.
9. If you can sell a chair for \$120 and a swing for \$100, then write an equation that gives your total revenue, dependent upon the number of chairs and swings that you sell in a week. ($R = \dots\dots\dots$)
10. Test your each of the (x,y) coordinates of the corner points found in #3, and determine which ordered pair gives you the greatest revenue.
11. Interpret your results: State how many chairs and how many swings you should make to maximize your revenue.
- 12.

Part II is on the other side.

PART II: Following a Structure

Just as in mathematics, in everyday life you are always making some decisions based upon some set of conditions. For this part, you are to think about one of your hobbies (or a few different hobbies) or your job, and examine how you follow at least 5 “rules”, or conventions, or procedures to accomplish your hobby or work.

**For each of your 5 rules, conventions, or procedures, think of a situation that requires you to apply the rule to take one action over another. Explain what decision you would make, why your rule would dictate this decision, and what would happen if you did not use the rule to make the correct decision.

Example of examining a rule:

After a slice into the trees, and then a few shots to get back out, I find myself about 180 yards from a pond. It is another 40 yards across the pond, so I would need to hit my golf ball over 210 yards in the air to clear the water. There is no wind. **I think about my procedure for choosing the correct golf club:** I know about how far I can hit my clubs, and 220 yards in the air with my 3 wood would be a gamble shot for my amateur golfing skills. However, I know that I am capable of it. I could also play it safe and hit a shorter club to land just before the water. **For this hole, my decision is to go for it and try to hit over the water.** If I use a shorter club, I am less likely to go in the water. However, since I am not playing for money, I will make the game more exciting and try to clear the water.

For my golf game, my procedure dictates that I use the club that would make a great golf shot if I hit the club correctly. I try to play as though I’m just a little better than I actually am. If I did not follow this procedure, golf wouldn’t be quite as much fun for me.

6. Sample Assessment

Survey of Algebra: Math 106**TEST 3
Chapter 5**

Name: _____ Date: _____

SHOW YOUR WORK where applicable. Label your answers. Check your answers. Please simplify all fraction answers. Unless otherwise specified, round any decimal answers to the nearest hundredth. CIRCLE OR BOX all answers. Good luck!

Each problem is worth 4 points.

1. Simplify: $((10x^6)/(-2x^2))$

2. Simplify: $-(-3)^2 \cdot 9^{-1}$

3. The diameter of a carbon atom is about 2.2×10^8 cm. A tiny diamond used on a concrete-cutting blade has a width of 4×10^2 cm. If the carbon atoms are lined up right next to each other, how many atoms does it take to span the width of the diamond? Give your answer in scientific notation.

4. Simplify: $((2x^3y^2)/(3xy^3))^{-1}$

5. Simplify: $[5.29 + (0.374)^{-1}]^2$

6. The formula $L = 64.3(1.0033)^a$ can be used to model life expectancy, L , for U.S. African-American males with present age a (National Center for Health Statistics). To what age can a 20-year-old African-American male expect to live?

7. Simplify: $(x^3-3x)-(1-x-2x^2)$

8. Multiply: $5x^2y^3(3x^2y-4x)$

9. Imagine that your annual bonus in dollars for selling n new projects from your engineering firm is given by the function:

$$B(n)=10n^2+120n-50$$

Determine your bonus for selling 15 new projects.

10. Multiply: $(3x^2-4)(2x^2+3)$

11. Multiply: $[(2x+3)+y][(2x+3)-y]$

12. Being new parents, we just got a big 8 inch by 12 inch picture of our son, Sam. My wife got a frame with borders of x inches wide. (The black edge in the figure is x inches wide.) Find a trinomial that can be used to represent the area of the entire picture and frame.



13. Factor completely: $8x^3-1$

14. Factor completely: $2a+2b+wa+wb$

15. Factor completely: $-3x^3-18x^2-27x$
16. Is the following a difference of squares, a difference of cubes, or both?
 x^6-y^{12}
Take just a first step at factoring this. You do not need to factor it completely.
17. Factor completely: x^2+8x-9 18. Factor completely: $2x^2-17x+21$
19. Factor completely: $9x^2+12x+4$ 20. Factor completely: $x^4-(2x+1)^2$
21. Factor completely: $(x^2-8)^2-4(x^2-8)-32$
22. Factor completely: x^3+25x
23. Solve: $2x^2+7x=15$ 24. Solve: $(x+3)(x^2-16)=0$

25. A boy shoots a stone out of a slingshot, and he notices that the height $h(t)$ of the stone in feet at time t in seconds can be modeled with the function

$$h(t) = -16t^2 + 80t.$$

For how long is the stone in the air?

EXTRACREDIT (4 points)

Given that $(x+2)^3 = x^3 + 6x^2 + 12x + 8$, determine the general pattern for a cube of a sum:
 $(a+b)^3 = ?$

For example, the general pattern for the square of a sum is $(a+b)^2 = a^2 + 2ab + b^2$.

Activities Supplement

Marginal Gloss Questions

The intention of the Marginal Gloss is to have students read their textbooks critically. Bear in mind, the questions below are specific to certain texts and pages in that text, but they serve as an example of what can be done. Each group is from one page of Marginal Gloss questions.

- What does it mean that $(x+1)$ is a factor?
- What do you know if the remainder is 0?
- Where did you get these factors?
- How do you factor the remaining trinomial?

- What is meant by clearing?
- Why did they use 6?
- Notice this was done on both sides.

- What is example 16?
- In $u^2 = d$, why is $d > 70$?
- Could you solve example 2 by factoring?
- Is one method easier for you than the other? (ex 1 versus ex 2?)

- Do the logarithms have to have the same base? Why?
- State the Quotient Rule in your own words.
- What is "ln"?
- Explain this step $\ln(w^8/w^2) = \ln(w^6)$

- Why did they insert 2 zeros when continuing to divide?
- Why does the divisor need to be a whole number?
- If we move the decimal, why does it not change the answer?

Important Elements of Any Lesson Plan

The items below were generated by faculty in response to the question, "What do you think are essential elements for any lesson plan?"

Group One:

1. Administrative details, questions, review.
2. Preview new concepts
 - make it practical
 - relate to previous and future concepts, applications, etc.
3. Introduce/present
 - simple
 - more difficult
 - incorporate review
 - variety
4. Individual or group or partner PRACTICE
5. Encourage interaction with the textbook
6. Make clear assignments with detailed expectations. "Show all work"

Group Two:

1. Transition from old to new
2. Overview (conceptualization of goals)
3. New Material → Practice and Example
4. Summary

Group Three:

1. Connect to some previous topic: Recall...
2. Give an overview of the topic, implications, importance,...
3. Present the new material
 - Examples
 - Group practice
4. Tell them what you told them... or have students summarize.
 - "Why do you need to know this?"
5. Now what? (Outside of class.)
 - Preview of coming attractions

Attention Control

“What do you do to control attention in your classroom?” Below are some responses from faculty.

1. Physical Activity
2. Group interaction
3. Tone of voice (volume)
4. Call on students by name

1. Face the classroom as often as possible.
2. Pause after presenting a problem
3. Consistent HW/quiz routine
4. Vary activity within class and from class to class

1. Be prepared (fore planning and thought)
“If I don’t have a plan, the students will.”
2. Intersperse new-topic explanations with practice problems for students
3. Eye contact
4. Personal contact – move through the room